

① $\lim_{x \rightarrow +\infty} \frac{3}{\sqrt{4x^2 - 2x} - 2x} + 5$

✗ Inoltre provare che se $f(x)$ è una funzione tale che s'ha

$$\frac{\sqrt{x}-2}{x-4} \leq f(x)$$

e

$$f(x) \leq \frac{1}{x}$$

$\forall x \neq 4$ in un fissato intervallo di ϵ

$\Rightarrow$ Allora $\lim_{x \rightarrow 4} f(x) = \frac{1}{4}$

② Dimostrare, ~~esistere~~ e commentare:

Siano $x_0 \in \mathbb{R}$ ed $f(x)$ una funzione.

Se $\lim_{x \rightarrow x_0} f(x)$ esiste allora esso è unico

③ $\int_0^{\frac{\pi}{2}} (2 + 3\sqrt{4 + 3\sin x}) \cos x \, dx$

④ Scrivere, se esiste, l'equazione della Tangente al grafico della funzione $f(x) = 3\cos \log x$ in $x_0 = e^{-\frac{1}{2}}$

⑤ $y = \frac{2}{\sqrt{16-x^2}} + 3$

$$\lim_{x \rightarrow +\infty} \frac{3}{\sqrt{4x^2 - 2x} - 2x} + 5 =$$

$\infty - \infty$ form

$$\lim_{x \rightarrow +\infty} \frac{3(\sqrt{4x^2 - 2x} + 2x)}{\sqrt{4x^2 - 2x} - 2x} + 5 = \lim_{x \rightarrow +\infty} \frac{3\left(\sqrt{x^2\left(4 - \frac{2}{x}\right)} + 2x\right)}{-2x} + 5 =$$

$$= \lim_{x \rightarrow +\infty} \frac{3\left(\sqrt{4x^2} \sqrt{4 - \frac{2}{x}} + 2x\right)}{-2x} + 5 = \lim_{x \rightarrow +\infty} \frac{3\left(|x| \sqrt{4 - \frac{2}{x}} + 2x\right)}{-2x} + 5 =$$

$$= \lim_{x \rightarrow +\infty} \frac{3\left(x \sqrt{4 - \frac{2}{x}} + 2x\right)}{-2x} + 5 = \lim_{x \rightarrow +\infty} \frac{3\left(\sqrt{4 - \frac{2}{x}} + 2\right)}{-2} + 5 =$$

$$= \lim_{x \rightarrow +\infty} \frac{3\left(\sqrt{4 - \frac{2}{x}} + 2\right)}{-2} + 5 = \lim_{x \rightarrow +\infty} \frac{3(2 + 2)}{-2} + 5 = -6 + 5 = -1 \checkmark$$

$$\frac{\sqrt{x}-2}{x-4} \leq f(x) \leq \frac{1}{x}$$

dimostrare che $\forall x \neq 4$ in un
~~per~~ finito intervallo di x

$$\Rightarrow \lim_{x \rightarrow 4} f(x) = \frac{1}{4}$$

$$\lim_{x \rightarrow 4} \frac{\sqrt{x}-2}{x-4} = \frac{\sqrt{4}-2}{4-4} = \frac{0}{0} \stackrel{H}{=} \frac{\frac{1}{2\sqrt{x}}}{1} = \frac{1}{2\sqrt{4}} = \frac{1}{4}$$

$$\lim_{x \rightarrow 4} \frac{1}{x} = \frac{1}{4}$$

quindi se

$$\frac{\sqrt{x}-2}{x-4} \leq f(x) \leq \frac{1}{x}$$

allora

$\frac{1}{4}$

$$\int_0^{\frac{\pi}{2}} (2 + 3\sqrt{4+3\cos x}) \cos x \, dx =$$

$$4 + 3\cos x = t$$

$$3\cos x \, dx = \frac{dt}{3}$$

$$= \int_0^{\frac{\pi}{2}} (2 + 3\sqrt{t}) \frac{dt}{3} = \int_0^{\frac{1}{2}} \frac{2}{3} + \sqrt{t} \, dt = \int_0^{\frac{1}{2}} \frac{2}{3} dt + \int_0^{\frac{1}{2}} \sqrt{t} \, dt =$$

$$= \left[\frac{2}{3}t + \frac{t^{\frac{1}{2}+1}}{\frac{1}{2}+1} \right]_0^{\frac{1}{2}} = \left[\frac{2}{3}t + \frac{t^{\frac{3}{2}}}{\frac{3}{2}} \right]_0^{\frac{1}{2}} = \left[\frac{2}{3}t + \frac{2}{3}\sqrt{t^3} \right]_0^{\frac{1}{2}} =$$

$$= \left[\frac{2}{3}(4+3\cos x) + \frac{2}{3}\sqrt{(4+3\cos x)^3} \right]_0^{\frac{\pi}{2}} =$$

VER. INT.

$$\frac{2}{3}(3\cos x) + \frac{2}{3} \cdot \frac{3}{2} (4+3\cos x)^{\frac{1}{2}} \cdot (3\cos x) =$$

$$= \cos x (2 + 3\sqrt{4+3\cos x}) \quad \checkmark$$

$$= \left(\frac{2}{3}(4+3\cos \frac{\pi}{2}) + \frac{2}{3}\sqrt{(4+3\cos \frac{\pi}{2})^3} \right) - \left(\frac{2}{3}(4+3\cos 0) + \frac{2}{3}\sqrt{(4+3\cos 0)^3} \right) =$$

$$= \frac{2}{3}(4+3) + \frac{2}{3}\sqrt{(4+3)^3} - \left(\frac{2}{3}(4+3 \cdot 0) + \frac{2}{3}\sqrt{(4+3 \cdot 0)^3} \right) =$$

$$= \frac{14}{3} + \frac{2}{3} \cdot 7\sqrt{7} - \left(\frac{2}{3} \cdot 4 + \frac{2}{3} 4\sqrt{4} \right) =$$

$$= \frac{14}{3} + \frac{14}{3}\sqrt{7} - \frac{8}{3} - \frac{16}{3} = 4,66 + 12,34 - 2,6 - 5,33 = 9,07 \approx 9,01$$

✓

$$y = 3 \cos \log x \quad x_0 = e^{-\pi/2}$$

$$y' = -3 \frac{\sin \log x}{x} \bigg|_{x_0 = e^{-\pi/2}} = -3 \frac{\sin \log (e^{-\pi/2})}{e^{-\pi/2}} =$$

$$= \frac{-3 \sin(-\frac{\pi}{2})}{e^{-\pi/2}} = \frac{-3 \cdot (-1)}{e^{-\pi/2}} = \frac{3}{e^{-\pi/2}} = \textcircled{m} = y'(x_0)$$

$$x_0 = e^{-\pi/2} \Rightarrow y_0 = 3 \cos \log e^{-\pi/2} = 3 \cdot \cos(-\frac{\pi}{2}) = 0$$

$$P(e^{-\pi/2}; 0)$$

$$\text{Fesdo per } P + \textcircled{\text{m Trovato}} \rightarrow m = y'(x_0)$$

$$y - y_0 = y'(x_0)(x - x_0)$$

$$y - 0 = \frac{3}{e^{-\pi/2}} (x - e^{-\pi/2}) \Rightarrow y = \frac{3}{e^{-\pi/2}} x - 3 \quad \text{VERIFICATO}$$

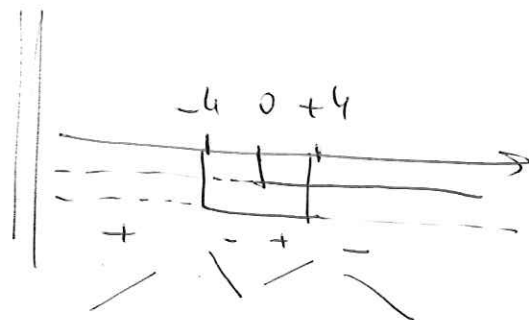
$$y' = \frac{-2 \cdot \frac{1}{2\sqrt{16-x^2}} \cdot (-2x)}{16-x^2} = \frac{2x}{\sqrt{16-x^2}(16-x^2)}$$

obs. $Dy' \equiv Dy$ porque derivamos $16-x^2 \neq 0$

Seja y' $2x \geq 0 \Rightarrow x \geq 0$

$$\sqrt{\quad} \geq 0 \quad \forall x$$

$$16-x^2 > 0$$



Solo $0 \in D \Rightarrow P.(0, \frac{7}{2})$ é mínimo

pro cálculo de int. ou y

$$y'' = \left(\frac{2x}{(16-x^2)^{3/2}} \right)' = \frac{2(16-x^2)^{3/2} - 2x \cdot \frac{3}{2}(16-x^2)^{1/2} \cdot (-2x)}{(16-x^2)^3}$$

$$= \frac{2\sqrt{(16-x^2)^3} + 6x^2\sqrt{16-x^2}}{(16-x^2)^3} = \frac{2\sqrt{16-x^2}((16-x^2)^{\frac{3}{2}-1} + 6x^2)}{(16-x^2)^3}$$

$$= 2(16-x^2)^{\frac{1}{2}-3}(16-x^2 + 6x^2) = 2(16-x^2)^{-\frac{5}{2}}(5x^2+16) =$$

$$= \frac{2(5x^2+16)}{\sqrt{(16-x^2)^5}} = \frac{2(5x^2+16)}{\sqrt{(16-x^2)^2} \sqrt{(16-x^2)^2} \sqrt{16-x^2}} = \frac{2(5x^2+16)}{(16-x^2)^2 \sqrt{16-x^2}} > 0$$

$$2 > 0 \quad \forall x$$

$$5x^2+16 \geq 0 \quad \forall x$$

$$(16-x^2)^2 \geq 0 \quad \forall x$$

$$\sqrt{16-x^2} > 0 \quad \forall x$$

